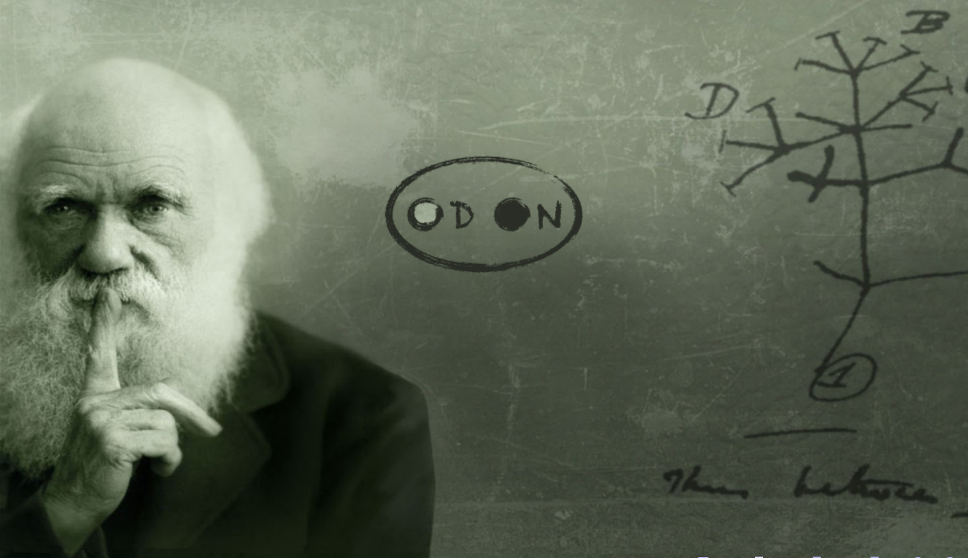
INTRODUCING

DARWINIAN NETWORKS



Introducing Darwinian Networks

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Outline of the Presentation

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2. Darwinian Networks

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    - Variable Elimination
    - Arc-Reversal
    - Lazy Propagation

    2.2 Modeling (Testing Independencies)
    - m-Separation
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3. Advantages

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1. Motivation

Understanding *Bayesian network* (BN) inference is not easy.

We sought a *purely graphical approach* for BN inference.

The representation took on a biological feel *(Darwinian Networks)*.

We then observed that Darwinian Networks could represent, simplify, and speed-up the testing of independencies.

And determine good elimination orderings.

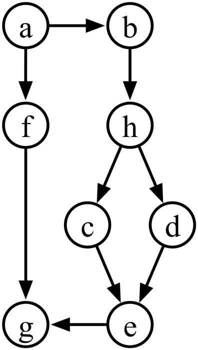
Surprisingly simple, remarkably robust.

Bayesian Networks

A *Bayesian Network* (BN) consists of:

- a *directed acyclic graph* (DAG),
- a matching set of *conditional probability tables* (CPTs).

Example: BN



$$P(U) = P(a) \cdot P(b|a) \cdot P(c|h) \cdot P(d|h) \cdot P(e|c,d) \cdots P(g|e,f)$$

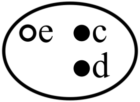
2. Darwinian Networks (DNs)

A CPT $P(X|Y)$ is represented as a *population* $p(X, Y)$.

The variables in the LHS X are *white*.

The variables in the RHS Y are *black*.

$$P(e|c,d) \Longrightarrow$$

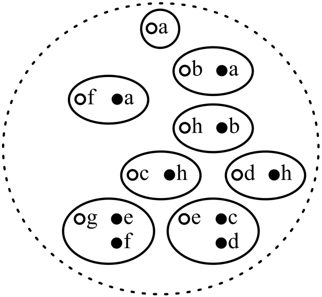


CPT $P(e|c,d)$ is depicted as population $p(e, cd)$.

2. Darwinian Networks

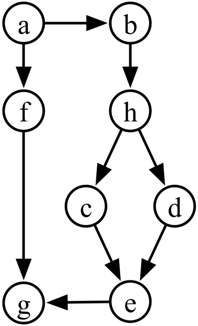
A *Darwinian Network* (DN) is a finite, multiset of populations.

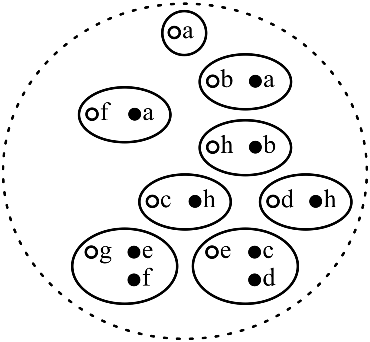
A DN is depicted by a dashed closed curve around its populations.



Populations:
$p(a)$, $p(b, a)$, $p(c, h)$, $p(d, h)$, $p(e, cd)$, $p(f, a)$, $p(h, b)$, $p(g, ef)$

Every BN can be represented as a DN





2.1 Inference

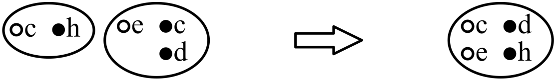
BN inference is called *evolution* in DNs.

We first introduce operations on populations corresponding to:

- multiplication,
- division,
- marginalization.

Multiplication is the Merge of Populations

$$P(c|h) \cdot P(e|c,d) = P(c,e|d,h)$$



○ white	+	● black	=	○ white
● black	+	○ white	=	○ white
● black	+	● black	=	● black
○ white	+	○ white	=	● black

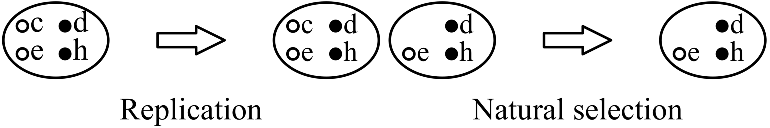
Merge also represents Division

$$P(a, b) \; / \; P(b) \quad = \quad P(a|b)$$

○ white	+	● black	=	○ white
● black	+	○ white	=	○ white
● black	+	● black	=	● black
○ white	+	○ white	=	● black

Marginalization is Replication then Natural Selection

$$\sum_c P(c, e|d, h) \quad = \quad P(e|d, h)$$



DNs can represent BN Inference Algorithms

As DNs can represent multiplication, division, and marginalization, it follows that DNs can represent exact inference algorithms, including:

- Variable Elimination (VE) (Zhang and Poole, 1994),
- Arc-Reversal (AR) (Olmsted, 1983),
- Lazy Propagation (LP) (Madsen and Jensen, 1999).

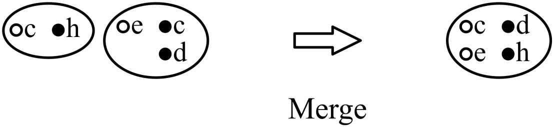
Variable Elimination (VE)

To answer $P(e|b)$, VE computes:

$$\begin{aligned}
P(c, e|d, h) &= P(c|h) \cdot P(e|c, d), && (1)\\
P(e|d, h) &= \sum_c P(c, e|d, h), && (2)\\
P(d, e|h) &= P(d|h) \cdot P(e|d, h), && (3)\\
P(e|h) &= \sum_d P(d, e|h), && (4)\\
P(e, h|b) &= P(h|b) \cdot P(e|h), && (5)\\
P(e|b) &= \sum_h P(e, h|b). && (6)
\end{aligned}$$

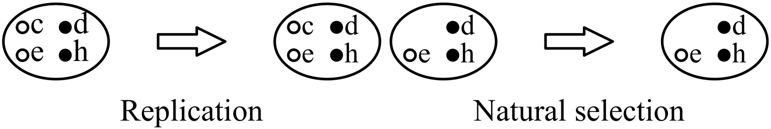
Example: Representing VE as DN Evolution

$$P(c|h) \cdot P(e|c,d) \quad = \quad P(c,e|d,h) \tag{1}$$



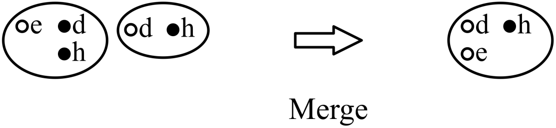
Example: Representing VE as DN Evolution

$$\sum_c P(c, e|d, h) \quad = \quad P(e|d, h) \tag{2}$$



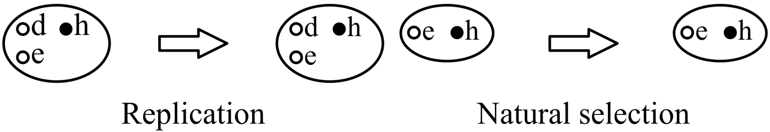
Example: Representing VE as DN Evolution

$$P(e|d,h) \cdot P(d|h) \quad = \quad P(d,e|h) \tag{3}$$



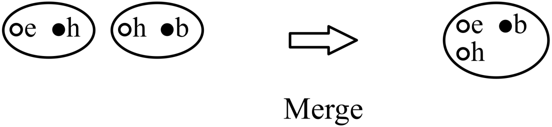
Example: Representing VE as DN Evolution

$$\sum_{d} P(d, e|h) \quad = \quad P(e|h) \qquad (4)$$



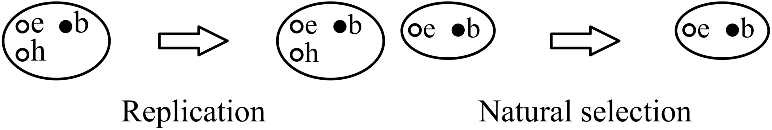
Example: Representing VE as DN Evolution

$$P(e|h) \cdot P(h|b) \quad = \quad P(e, h|b) \tag{5}$$



Example: Representing VE as DN Evolution

$$\sum_h P(e, h|b) = P(e|b) \quad (6)$$



Query $P(e|b)$ is represented as population $p(e, b)$.

DN Advantage

Koller and Friedman (2009) introduce readers to BN inference using VE.

There is a one-to-one correspondence between VE's mathematical equations and the DN illustrations.

Hence, one can learn VE without involving a single equation.

DNs can represent Arc-Reversal (AR)

AR eliminates a variable v_i by reversing the arc (v_i, v_j) between v_i and each child v_j of v_i.

$$\begin{aligned} P(v_i, v_j | P_i P_j) &= P(v_i|P_i) \cdot P(v_j|P_j), \\ P(v_j|P_i P_j) &= \sum_{v_i} P(v_i, v_j|P_i P_j), \\ P(v_i|P_i P_j v_j) &= \frac{P(v_i, v_j|P_i P_j)}{P(v_j|P_i P_j)}. \end{aligned}$$

AR only involves multiplication, division, and marginalization.

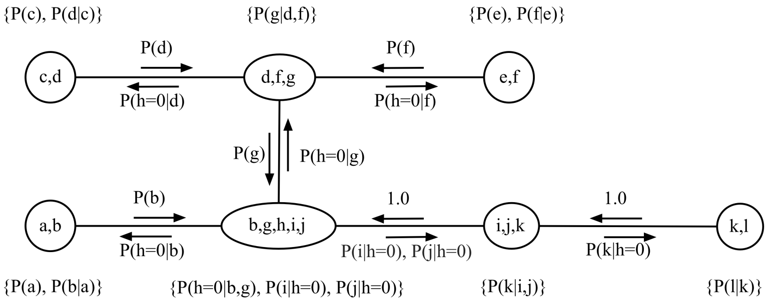
Lazy Propagation (LP)

BN variables are clustered into nodes, organized as a join tree.

Each BN CPT is assigned to a join tree node.

Messages are propagated systematically.

LP only involves multiplication, division, and marginalization.

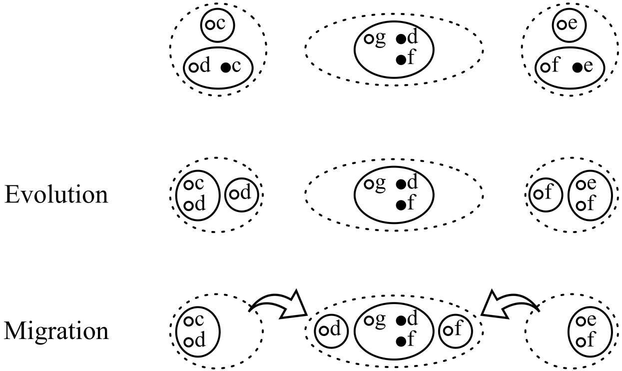


DNs can represent LP

A join tree is represented as a set of DNs.

Each join tree node is represented as one DN.

A propagated message from a join tree node to another is viewed as a population *migrating* from one DN to another.



2.2 Modeling - Testing Independencies in BNs

m-Separation (Lauritzen et al., 1990; Zhang and Poole, 1994) tests $I(X, Y, Z)$ in an undirected graph with four steps:

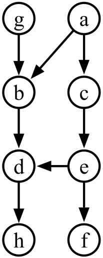
(i) construct the sub-DAG onto $XYZ \cup An(XYZ)$;

(ii) construct the *moralization* by adding an undirected edge between each pair of parents of a common child and then dropping directionality;

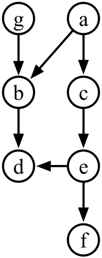
(iii) delete Y and its incident edges;

(iv) if there exists a path from X to Z, then $I(X, Y, Z)$ does not hold; otherwise, $I(X, Y, Z)$ holds.

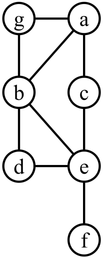
Example: m-Separation $I(a, d, f)$



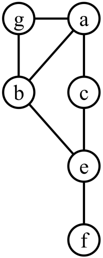
DAG



sub-DAG



moralization



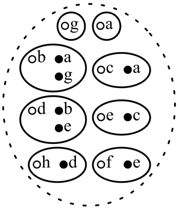
deletion
and
check for path

DNs can represent m-Separation

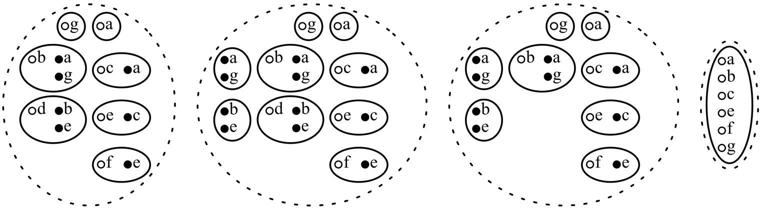
Testing independencies in BNs is represented as testing *adaptation* in DNs.

We observe how populations *adapt* to the removal of other populations.

Adaptation: Testing Independence $I(a, d, f)$ in DNs



(i) (ii) (iii) (iv)



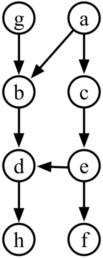
sub-DAG moralization deletion check for path

DNs Simplify m-Separation

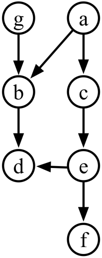
Moralization of m-separation can be excessive.

Adding edge (b, e) is necessary, since d will be deleted.

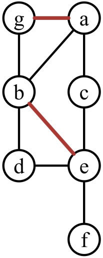
Adding edge (a, g) is unnecessary, since b will not be deleted.



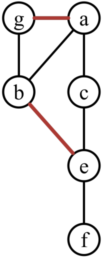
DAG



sub-DAG



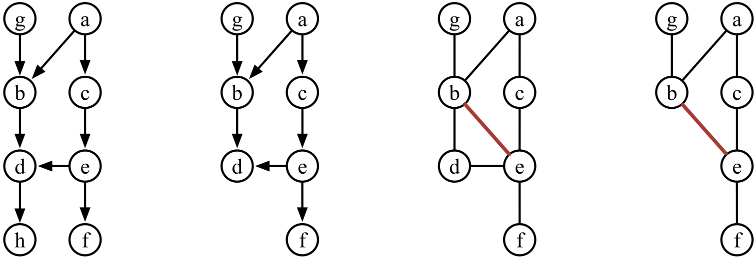
moralization



deletion
and
check for path

DN Contribution (Rationalization)

When testing $I(X, Y, Z)$, add an undirected edge between variables with a common child only when the child is in Y.



DAG

sub-DAG

rationalization

deletion
and
check for path

Modeling - Testing Independencies with d-Separation

Algorithm 3.1 Find nodes reachable from X given Y via active paths in DAG $\mathcal{B}$

```
procedure REACHABLE(X,Y,B)
    ▷ Phase I: insert Y and all ancestors of Y into A
    An(Y) ← ANCESTORS(Y, B)
    A ← An(Y) ∪ Y
    ▷ Phase II: traverse active paths starting from X
    for v ∈ X do                 ▷ (Node,direction) to be visited
        L ← L ∪ {(↑, v)}
    V ← ∅                        ▷ (Node,direction) marked as visited
    R ← ∅                        ▷ Nodes reachable via active path
    while L ≠ ∅ do               ▷ While variables to be checked
        Select (d, v) in L
        L ← L − {(d, v)}
        if (d, v) ∉ V then
            if v ∉ Y then
                R ← R ∪ {v}                ▷ v is reachable
            V ← V ∪ {(d, v)}  ▷ Mark (d, v) as visited
            if d =↑ and v ∉ Y then
                for v_i ∈ Pa(v) do
                    L ← L ∪ {(↑, v_i)}
                for v_i ∈ Ch(v) do
                    L ← L ∪ {(↓, v_i)}
            else if d =↓ then
                if v ∉ Y then
                    for v_i ∈ Ch(v) do
                        L ← L ∪ {(↓, v_i)}
                if v ∈ A then
                    for v_i ∈ Pa(v) do
                        L ← L ∪ {(↑, v_i)}
    return R
```

Geiger et al. (1989) provide a linear time complexity algorithm for implementing d-separation.

Implementation of d-separation given by Koller and Friedman (2009).

DNs Simplify d-Separation

The main idea is to start from X, follow active paths, and see if it reaches Z.

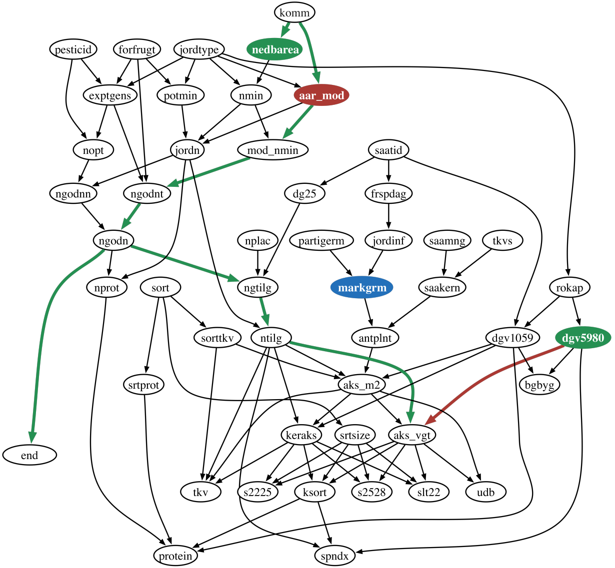
The linear implementation of d-separation considers all active paths until they become blocked.

The DN improvement is the identification of a class of active paths that are doomed to become blocked.

Thus, there is no benefit to exploring these paths.

Example: Testing $I(nedbarea, markgrm, dgv5980)$ in Barley

Any path through *aar_mod* will eventually become blocked.



DN Contribution

Stop traversing active paths that are doomed to become blocked.

BN	$\|N\|$	d-Sep Tests	i-Sep Tests	Test Savings	Time Savings
Insurance	27	58898	36642	38%	-22%
Water	32	41392	23959	42%	-27%
Alarm	37	35224	23078	34%	-11%
Barley	48	78794	56804	28%	-15%
Hailfinder	56	51922	42543	18%	-23%
Pathfinder	135	125932	62820	50%	-79%
Munin1	186	167809	64329	62%	14%
Diabetes	413	827291	681468	18%	11%
Pigs	441	116795	12841	89%	36%
Link	724	336780	75505	78%	38%
Munin4	1038	509299	77314	85%	47%
Munin3	1041	459409	50147	89%	55%

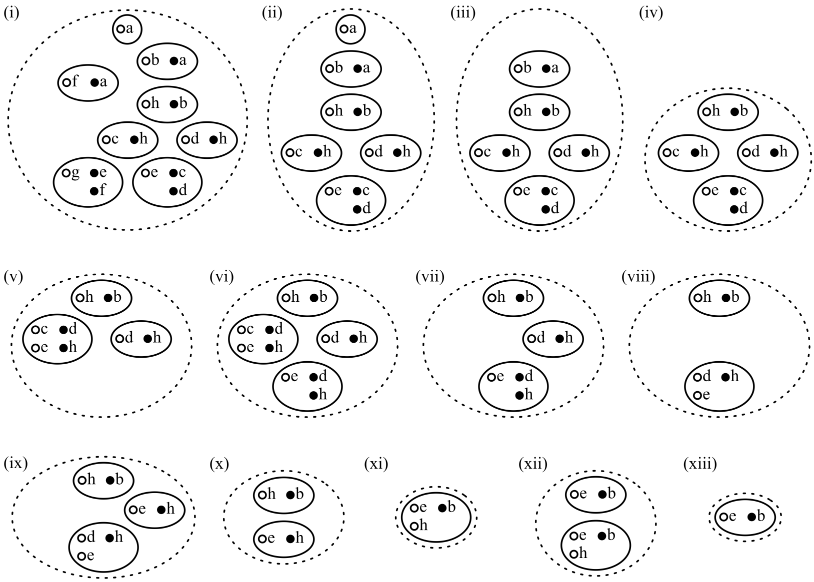
3. Advantages - Surprisingly Simple, Remarkably Robust

- In modeling, d-separation can use specialized terminology not referenced in inference such as "open sequential valves" and "closed divergent valves."
- In inference, LP involves specialized terminology not referenced in modeling such as the "running intersection property."
- DNs use the same terminology for inference and modeling.

Representing All Steps of LP in DNs

- LP involves 2 networks.
- LP tests independencies in a BN, yet conducts inference in a JT.
- DNs do both in the same network.

All Steps of VE: BN, query, independencies, computation



Determining Good Elimination Orderings

- The order in which variables are eliminated can have a profound impact on the amount of computation performed.

- DNs can represent four well-known heuristics for determining good elimination orderings in BNs:
  - *min-neighbours* (MN),
  - *min-weight* (MW),
  - *min-fill* (MF),
  - *weighted-min-fill* (WMF).

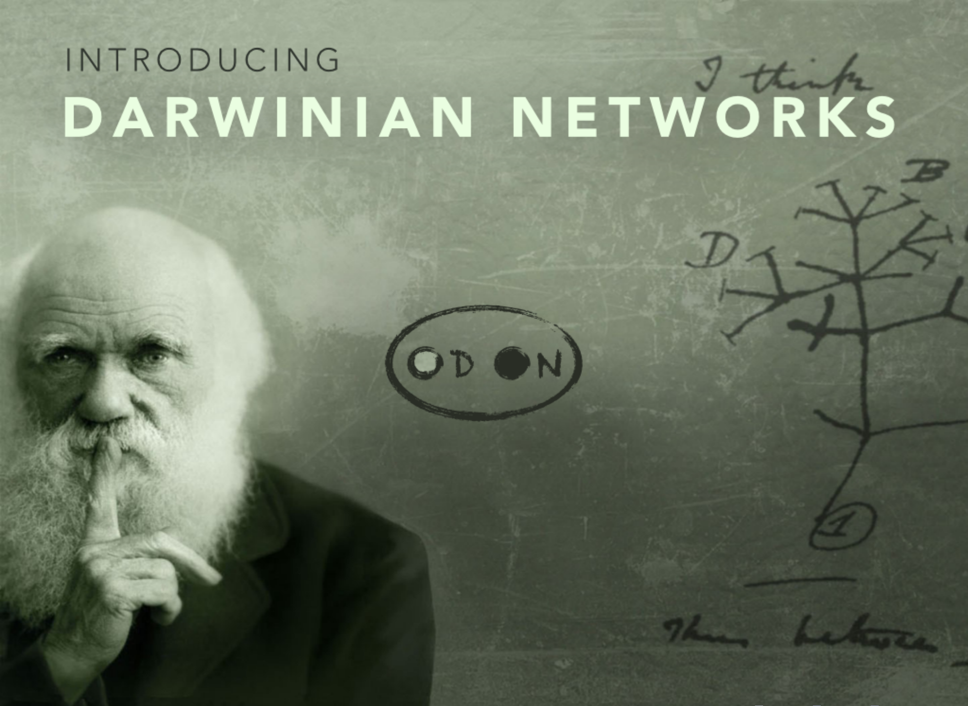
- We introduced a new heuristic, called *potential energy* (PE), based on DNs.

- PE can score more accurately than the above heuristics.

4. Conclusion

- Purely graphical approach to VE, which is often used to introduce BN inference to beginners
- Faster way to test independencies in BNs
- Unify modeling and inference into one network using common terminology
- DNs are like looking at BNs through a microscope

Watch www.darwiniannetworks.com for updates



Kilroy, Jeff. Darwin "I think..." 2009. Kilrizzy's deviantART gallery. deviantART. 2015. Web. 23 Apr. 2015.